2019-05-01

Platooning Control for Heterogeneous Sailboats Based on Constant Time Headway

viel, christophe

http://hdl.handle.net/10026.1/13771

10.1109/tits.2019.2912389
IEEE Transactions on Intelligent Transportation Systems
Institute of Electrical and Electronics Engineers (IEEE)

Platooning control for heterogeneous sailboats based on constant time headway

VIEL Christophe*, VAUTIER Ulysse*, WAN Jian*, JAULIN Luc**
**School of Engineering*, *University of Plymouth* , Plymouth, UK (name.surname@plymouth.ac.uk)
***Lab-STICC*, *ENSTA-Bretagne*, Brest, France (namesurname@gmail.com)

***Abstract*—This paper addresses the problem of platooning control for a fleet of heterogeneous sailboats. Platooning maintains a constant time headway (CTH) between sailboats following a circular path, a complex problem for sailboats due to the influence of wind direction. First, the desired acceleration based on the CTH and the sailboat velocity needed to converge to the platooning is defined. Second, a control of sailboat orientation to manage the sailboat acceleration is proposed. The proposed platooning strategy adapts to the specific characteristics of sailboats, which are different from other motorized marine vehicles. Two tack strategies can be used for the method: the first is to regulate the sailboat velocity; the second is to go front the wind, while staying in a short corridor. Desired acceleration for fulfilling the platooning has been derived and validated. Simulation results demonstrate the effectiveness of the proposed approach, with comparison to an optimal receding horizon control algorithm.**

***Index Terms*—Platooning, autonomous sailboats, non-linear control, heterogeneous fleet.**

I. Introduction

Over the past decades, there is a growing interest in marine robotics due to the needs for various marine research and exploration activities such as oceanography. Such robots can be used to transport materials between harbours and/or offshore stations, to find ship wreckages, to collect ocean floor data, and to conduct biological measurement or surveillance. Autonomous sailboats can play a significant role in such marine activities, particularly for long-duration missions because of their use of ocean renewable energy as their primary propulsion. However, autonomous robotic sailing is restricted by wind direction. The control task becomes very challenging with coupled wind conditions and complex sailboat dynamics.

The dynamics of a sailboat is studied in [18, 19, 25, 30, 3], represented by Euler-Lagrange dynamics or state-space equations. In these papers, an accurate estimation of the sailboat behavior is obtained by considering forces on the sail, keel and rudder. However, a perfect knowledge of the dynamic parameters for a real sailboat is difficult to obtain in practice and complex to implement on a real sailboat. That is why in some work like [27, 10, 23, 12], alternative methods like fuzzy methods or geometric laws are used to control rudder and sail without using dynamic models. The idea is to imitate common sailing practice, without knowing the sailboat dynamics. The main advantage of this technique is its applicability to every type of sailboats although the control performance may not be optimal. This paper follows these ideas to imitate sailing practice with knowing a very small number of sailboat dynamics parameters, which makes it easy to implement for various types of sailboats.

Studying the platooning problem for sailboats is interesting in terms of synchronizing boats for various missions such as ocean transportation, scanning, mapping and surveillance using a convoy of sailboats. A convoy of sailboats can be used to deliver supplies to remote islands or offshore infrastructures with energy efficiency and zero emission. It can also be used to conduct a 360-degree surveillance in coastal borders. All these tasks requires the sailboats to maintain certain formations and to avoid collisions. A circular platooning can therefore be used to maintain the fleet close to a target position or an observation area with better collision avoidance management. Moreover, another interest is to combine arcs and straight lines to define a platooning on a general reference trajectory.

Created to avoid traffic jam, a platooning allows to maintain a desired distance among vehicles. Over the past years, platooning studies mostly focus on ground vehicles, mainly studied in a very theoretical way like an optimization or a consensus problem like in [24, 17], detection of obstacle and choice of sensors like in [16, 1, 5], or problem of communication like communication delay and reduction of communication using event-triggered methods like in [7, 15, 6, 31]. However, platooning is more delicate for sailboats due to wind direction and the challenge to control the sailboat velocity exactly. As a diamond trajectory or a line allows sailboats to move upwind so as to get a similar wind orientation, platooning becomes rather difficult to obtain in case of a circular path where the wind direction changes for each boat. While ground vehicles, AUVs and classic surface vehicles can control their position/velocity exactly by using motors, exact sailboats positions/velocities are uncontrollable, making previous classic platooning strategies unsuitable to implement or follow straightforwardly. Moreover, some work like [9], where the main focus is to improve energy efficiency and eco-driving is not relevant to sailboats since sailboats use renewable energy to move.

Distance between two vehicles can be fixed as in [6, 2] or variable with vehicle velocity as in [4]. In the later case, a constant time headway (CTH) is defined, corresponding to the desired time that a vehicle needs to reach the position of its predecessor. The time headway, developed in [4], allows to respect a security distance proportional to vehicles velocity, for example for cars on highway to avoid collision in case of sudden braking. CTH is really interesting for the sailboat platooning problem due to the difficulty to control sailboat breaking: it allows to stay the closest to the other sailboat without risk of collision. Moreover, since sailboats require a considerable velocity to be controlled, a constant time headway is mostly desirable than a fixed distance for allowing sailboats to move at their maximal velocity.

Platooning problems for surface and underwater vehicles have been studied in [14, 13]. In these work, a distributed RHC

(Receding Horizon Control) problem for nonlinear vehicle platooning with input and state constraints is developed, where an optimization allows keeping a stable platooning formation. Similar work for ground vehicles can be found in [31, 24], which consider a robustness analysis and distributed H-infinity controller for a platooning of connected vehicles with undirected topologies and presence of external disturbances. Optimization problems are formulated to obtain an optimal undirected topology for a platoon system. These sub-platooning tasks are linked by the presence of a leader at the top of the fleet. However, methods based on general nonlinear dynamic models can not obtain efficient results compared to methods developed for a specific model, mostly for vehicles with very specific dynamics like a sailboat. Moreover, since in real-world applications, some information can be difficult to obtain due to disturbances like wind, wave and current conditions, making it difficult to implement. That is why neurodynamics observers are proposed in [21, 20] to recover the unmeasured velocity information and unknown vessel dynamics. Similarly, neurodynamic optimization and fussy approximation are used in [22] to evaluate unknown dynamics parameters. Control law of platooning is constructed based on the estimated parameters and an optimal guidance signal, shared by all vessels and obtained by the neurodynamic optimization.

In this paper, a platoon strategy adapted to sailboats is proposed, allowing better results than a method with general nonlinear dynamics model like in [14, 13], and without the need of complex dynamics parameters as required in [18, 19]. The proposed approach is compared with the optimal receding horizon control (RHC) algorithm exposed in [13] for showing its advantages. Thus, the main contributions of this paper are as follows:

- a platooning method to follow a linear and circular trajectory and it is adapted to an heterogeneous fleet of sailboats and their specific constraints.
- a control of sailboat orientation to manage the sailboat acceleration. This one allows performing a proposed platooning strategy, adapted to sailboat' specific dynamics, different from other marine vehicles.
- a simple method independent of sailboat dynamics to be applicable to various kinds of sailboats. Only few sailboat parameters are required to implement it.

To our knowledge, no other paper addresses the problem of platooning for sailboats so far. Distance between sailboats is defined using a CTH like in [4] to adapt security distance between sailboats and to move at their maximum velocity. A tack strategy has been created to regulate the projection of the sailboat acceleration and to go front the wind. Control methods for the sail angle and rudder are explored. Finally, an application example, consisting of the control of a sailboat fleet, is simulated so as to demonstrate the proposed approach.

The outline of the paper is as follows. Problem statement of platooning and sailboat parameters are presented in Section II-A, Section II-B and Section II-C, respectively. Specific constraints on the platooning are exposed in Section II-D. Parameters of the platooning trajectory are described in Section III. Control of platooning is described in Section IV. Evaluation of the desired acceleration is exposed in Section IV-A, and a tack strategy to regulate sailboat velocity and to define sailboat orientation is proposed in Section IV-B. A second tack strategy to move upwind is also described in Section IV-B2, with comparison to the first tack strategy. Low level control of rudder and sail are defined Section V-A and Section V-B, respectively. Section VI presents some simulation results and Section VII concludes the paper.

II. Problem description

A. Sailboat parameters

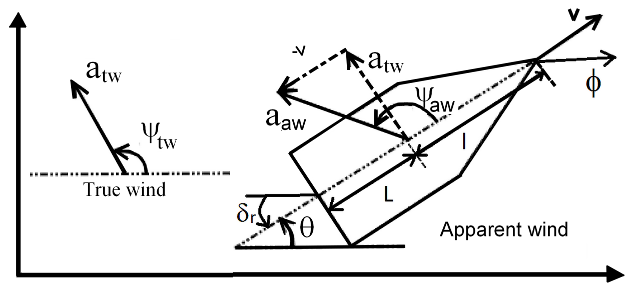


Figure 1: [18] Fixed distance parameters.

The notations used in this paper are described in Table I.

θ	orientation of the sailboat
v	velocity of the sailboat
ω	rotation speed
ϕ	course angle
δ_r	angle of rudder, $\delta_r \leq \delta_{r,\max}$
δ_s	angle of sail, $\lvert\delta_s\rvert \leq \delta_{s,\max}$
$\psi_{tw}(t)$, $a_{tw}(t)$	direction and speed of the true wind, described in Section II-B.
$\psi_{aw}(t)$, $a_{aw}(t)$	direction and speed of the apparent wind, described in Section II-B,
δ	hauled angle which define the dead area $\Omega_d(t) = [\psi_{tw}(t) + \pi - \delta, \psi_{tw}(t) + \pi + \delta]$,
M_i	sailboat i mass.

Table I: Sailboat parameters.

The wind direction and speed are not assumed to be constant in this paper. The notation ψ_{tw}, a_{tw}, ψ_{aw}, a_{aw} represents $\psi_{tw}(t)$, $a_{tw}(t)$, $\psi_{aw}(t)$, $a_{aw}(t)$. The strategies developed in this paper are defined for the current wind conditions at instant t and they can be updated new wind conditions.

Sailboats are assumed to be equipped with the following sensors:

- a weather vane and a wind gauge measure ψ_{wv} and a_{wv} such the apparent wind direction $\psi_{aw} = \pi - \psi_{wv}$ and velocity, $a_{aw} = a_{wv}$,
- GPS sensor to measure sailboat position p_S.
- IMU or GPS to evaluate the sailboat velocities v and u.
- a compass to measure sailboat heading θ, where $\theta = 0$ corresponds to the north.

As shown in [26], the dynamics of a sailboat i can be expressed with the general form

$$M_i \dot{v}_i = \tau_{f,i} + \tau_{r,i} + \tau_{s,i} + \tau_{w,i} \quad (1)$$

$$J_i \ddot{\phi}_i = C_{r,i} + C_{s,i} + C_{k,i} + C_{h,i}, \quad (2)$$

where J_i is the sailboat inertia matrix, ϕ_i is the course angle of the sailboat i, $\tau_{f,i}$, $\tau_{r,i}$, $\tau_{s,i}$, $\tau_{w,i}$ is the action of friction/drift, rudder, sail and wind and $C_{r,i}, C_{s,i}, C_{k,i}, C_{h,i}$ are the rudder, sail, keel and hydrodynamic actions. Since a perfect knowledge of the dynamic parameters of the sailboat is difficult to obtain in practice, the proposed algorithm and control scheme in this paper are independent of the system (1)-(2) and use those parameters that are simple to measure like in [27, 10, 23, 12]. Thus, they can adapt to different models of sailboats (different sizes of mono-hull, catamarans) by using the same control of the sail and the rudder exposed in Section V.

B. True and apparent wind

As illustrated in Figure 1, let the True wind (tw) be the wind measured from a fixed referential, and Apparent wind (aw) be the wind measured by a weather vane on a moving referential. The orientation $\psi_{tw} = 0$ corresponds to the north, and $\psi_{aw} = 0$ points the sailboat heading, such $\psi_{tw} = \psi_{aw} + \theta$ if sailboat is static, *i.e.* $v = 0$ in (3)-(4).

As exposed in [18], apparent wind can be evaluated in polar coordinate such

$$\left\{ \begin{array}{c} a_{aw} = \|A_w\| \\ \psi_{aw} = \text{atan2}\,(A_w) \end{array} \right., A_w = \left[\begin{array}{c} a_{tw}\cos(\psi_{tw} - \theta) - |v| \\ a_{tw}\sin(\psi_{tw} - \theta) \end{array} \right], \quad (3)$$

with A_w is the apparent wind in the Cartesian coordinate.

In the same way, an estimation of the true wind can be obtained from apparent wind :

$$\left\{ \begin{array}{c} a_{tw} = \|T_w\| \\ \psi_{tw} = \text{atan2}\,(T_w) \end{array} \right., T_w = \left[\begin{array}{c} v\sin(\phi) + a_{aw}\sin(\psi_{aw} + \theta) \\ v\cos(\phi) + a_{aw}\cos(\psi_{aw} + \theta) \end{array} \right], \quad (4)$$

where T_w is the apparent wind in the Cartesian coordinate.

The sailboat is back wind if $\cos(\psi_{tw} - \theta) > 0$, and upwind if $\cos(\psi_{tw} - \theta) + \cos(\delta) < 0$. In other scenarios, the sailboat is crosswinds, which is the default configuration.

C. Problem statement

Consider the group of N heterogeneous sailboats is moving along one or a combination of the following paths:

- a circle $\mathcal{C}(p_c, R)$ of center $p_c = [x_c, y_c]$ and radius R.
- a line AB defined by two points $A = [x_A, y_A]$ and $B = [x_B, y_B]$. Let $\widehat{AB}$ be the angle between points A and B.

Sailboats must stay inside a corridor of length $2r_{secu}$, *i.e.*

- if the path is circular, two circles with the interior circle $\mathcal{C}(p_c, R_{int})$ and the exterior circle $\mathcal{C}(p_c, R_{ext})$ where $R_{ext} = R + r$ and $R_{int} = R - r_{secu}$.
- if the path is linear, two lines $\bar{\text{AB}}$ and $\underline{\text{AB}}$ defined by points $\bar{\text{A}} = A + \bar{r}$, $\bar{\text{B}} = B + \bar{r}$ and $\underline{\text{A}} = A - \bar{r}$, $\underline{\text{B}} = B - \bar{r}$ where $\bar{r} = r_{secu}\left[\cos\left(\widehat{AB} + \frac{\pi}{2}\right), \sin\left(\widehat{AB} + \frac{\pi}{2}\right)\right]$.

Combination of circles and lines can be used to define the reference trajectory.

The sailboat i is noted S_i with the Cartesian position $p_i = [x_i, y_i]^T$, corresponding to the sailboat mast. The sailboat in front of S_i is S_j, where S_j is S_{i-1} for $i \in [2, \ldots N]$ and S_N is the sailboat in front of S_1 if sailboats are in a closed-loop path. Else, S_1 follows a virtual leader with the fixed velocity v_0. Suppose each sailboat i can receive messages from sailboat j, *i.e.* the sailboat in the front, and can transmit message to the sailboat behind. Since the dynamics of the sailboat is much slower than the communication time, the communication delay is considered to be negligible. Each sailboat i transmits its current position p_i, its heading θ_i, its velocity v_i and acceleration $\dot{v}_i$.

Let d_{ij} be the projection of the distance between sailboat i and j on the circle $\mathcal{C}(p_c, R)$ or the line AB. Each sailboat i tries to maintain the desired spacing distance d^*_{ij} with sailboat j. The desired spacing distance is defined such that $d^*_{ij} = d^*_{ij} = V_i T^* + d_{secu}$, where T^* is the constant *time headway*, *e.g.*, the time that the i-th sailboat takes to arrive at the position of sailboat j, $d_{secu} > 0$ a constant security distance (to avoid sailboats to crash each other if $V_i = 0$), and V_i is the projection of the velocity v_i on the reference trajectory, *i.e.* the circle $\mathcal{C}(p_c, R)$ or the line AB. Note d^*_{ij} is variable with the sailboat velocity, inducing d^*_{ij} is large when vehicles move fast and small when they are slow. Since sailboats are vehicles which require a good velocity to be controlled, all sailboats try to move at their maximal velocity. However, the sailboat maximum velocity is variable with the wind direction, and so it cannot be identical for all sailboats if they follow a circular path. Hence a constant time headway (CTH) to manage the distance between sailboats and not a fixed distance with a fixed velocity. In the case where sailboats follow a linear path, CTH becomes a constant distance policy.

D. Platooning parameter constraint

In this study, heterogeneous sailboats can be controlled inside the fleet. Due to the number of sailboats, their different weight and size, some constraints must be respected to guarantee the feasibility of a platooning, exposed in this section.

Let L_i be the length between the rudder and the keel of sailboat i and l_i be the length between the keel and the bow. The turning radius of the sailboat i is defined by $r_{r,i} = \frac{L_i}{\sin(\delta_{r,\max})}$, where the turning radius is a non-holonomic constraints describing the smallest circle $\mathcal{C}(p_i, r_{r,i})$ which can not perform by the sailboat i.

To avoid collision with the other sailboat, two points must be considered:

- A sailboat can stop only if it is back wind, *i.e.* $\cos(\psi_{tw} - \theta_i) > 0$. Let $d^{(1)}_{secu}$ be the turning distance to place sailboat upwind or perform a maneuver to arrive upwind. As described in Section VIII-B, $d^{(1)}_{secu}$ can be expressed such
$$d^{(1)}_{secu} = 3\max_{i=1:N}(r_{r,i}) + \max_{i=1:N}(l_i) + \max_{j=1:N}(L_j). \quad (5)$$
- To decelerate from the sailboat i maximal velocity $v_{i,\max}$ to 0, a stopping distance note $d^{(2)}_{secu}$ must be maintained between the sailboat and the one in front of it. As described in Section VIII-C, $d^{(2)}_{secu}$ can be expressed such
$$d^{(2)}_{secu} = \max_{i=1:N}\left(\frac{M_i}{C_i}v_{i,\max}\right) + \max_{i=1:N}(l_i) + \max_{j=1:N}(L_j), \quad (6)$$
where M_i is sailboat i mass and C_i is the tangential friction of sailboat i, whose measurement is described at the end of this section.

Based on $d^{(1)}_{secu}$ and $d^{(2)}_{secu}$, let define the condition on d_{secu}:

$$d_{secu} \geq \max\left(d^{(1)}_{secu}, d^{(2)}_{secu}\right). \quad (7)$$

In the same way and to implement strategy to be exposed in Section IV-B, we desire the sailboat velocity can converge to zero when it moves perpendicularly to the reference circle $\mathcal{C}(p_c, R)$ or line AB. Thus, take $r_{secu} \geq d_{secu}^{(2)}$.

The conditions on the interior circle R_{int} are defined as follows. The radium of the smaller circle performed by the sailboat i is $r_{r,i}$. Moreover, the sailboat needs to maintain a distance d_{ij}^* with the sailboat in front of them, where $d_{secu} \leq d_{ij}^* \leq T^* v_{\max} + d_{secu}$, and $v_{\max} = \max_{i=1:N} v_{i,\max}$. Thus the interior circle of the corridor must be a parameter such

$$R_{int} > \frac{N}{2\pi}\left(T^* v_{\max} + d_{secu}\right). \tag{8}$$

From these conditions, one obtains $R = R_{int} + r_{secu}$ and $R_{ext} = R + r_{secu}$. Note d_{secu}, R_{int} and R_{ext} are evaluated using parameters of all sailboats so as to consider their heterogeneity.

Measurement of C_i: Due to the hypothesis made in Appendix VIII-C on the sailboat dynamics, value of C_i can be measured in practice. This one may not be close to the true value, but it is the adapted value for our method. For that, consider that the sailboat is moving at the instant $t = 0$ such $v_i(0) = v_{ini}$ and $\theta_i = \psi_{tw} + \frac{\pi}{2}$. At instant $t = 0^+$, take $|\delta_s| = \frac{\pi}{2}$, *i.e.* open the sail completely until wind is not a driving force anymore, and measure the stopping distance $d_{stop,i}$ between $t = 0^+$ and instant $t = t_f$ where $v_i(t_f) = 0$. One can evaluate C_i such $C_i = \frac{M_i}{d_{stop,i}} v(0)$.

Justifications of the dynamic model used for the measurement of C_i are the same to the ones used for the evaluation of d_{secu}, and they can be found in Appendix VIII-C.

III. Platooning path parameters

In this paper, a platooning can be performed by following two kinds of paths: line or circle. Parameters linked to these trajectories are described in the following section, and used in Section IV to define the control of sailboats. More complex trajectories for platooning can be made by the combination of circles and lines.

A. Linear platooning

Due to physical constraints or wind direction, a linear path or a diamond trajectory allows to avoid the dead zone of the wind so as to obtain a smoother control of the sailboat.

If sailboats follow a linear trajectory, the following notations are defined:

- $\theta_i^* = \widehat{AB}$ is the default sailboat heading, allowing to follow the line AB.
- $V_i = v_i \cos(\theta_i^* - \theta_i)$ is the projection of the velocity of sailboat i on the line AB. In the same way, take $\dot{V}_i = \dot{v}_i \cos(\theta_i^* - \theta_i)$.
- $e_i = \det\left(\frac{B-A}{\|B-A\|}, p_i - A\right)$ is the distance of the boat to the line AB.
- q_i is the tack variable.

The distance d_{ij} between sailboat i and j can be evaluated as

$$d_{ij} = \bar{d}_{ij}\left|\cos\left(\widehat{\mathrm{IJ}} - \widehat{AB}\right)\right|, \tag{9}$$

where $\widehat{\mathrm{IJ}}$ is the angle between sailboat i and j, and $\bar{d}_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$.

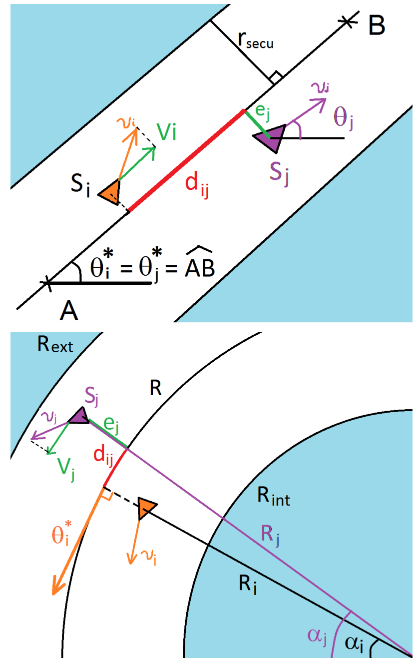


Figure 2: Top: Parameters for platooning with a linear path. Bottom: Parameters for platooning with a linear path..

These notations are illustrated in Figure 2 and are to be used in Section IV-A and IV-B to define the desired acceleration and heading.

B. Circular platooning

In a classic closed-loop platooning, turning around a circle is possible for a land vehicle, a plane or a drone, because the vehicle's velocity is independent of its orientation. In case of sailboats, wind direction creates a dead zone where boats can not move easily. Defining a diamond route allows sailboats to avoid dead area, but here we propose a method to follow a circular path.

If sailboats follow a circular trajectory, the following notations are defined:

- $\alpha_i = \text{angle}(x_i - x_c + 1i(y_i - y_c))$ is the angle of sailboat i around the circle $\mathcal{C}(p_c, R)$.
- s is the direction of the rotation of sailboats around the circle ($s = 1$ for the trigonometric direction, $s = -1$ for clockwise).
- $\theta_i^* = \alpha_i + \frac{\pi}{2}s$ is the default sailboat heading, allowing to follow the circular path.
- $R_i = \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2}$ is the distance between the center p_c of the circle $\mathcal{C}(p_c, R)$ and the sailboat i.
- $V_i = v_i \cos(\theta_i^* - \theta_i)\frac{R}{R_i}$ is the angular velocity of sailboat i around the circle $\mathcal{C}(p_c, R)$. In the same way, take $\dot{V}_i = \dot{v}_i \cos(\theta_i^* - \theta_i)\frac{R}{R_i}$.
- $e_i = R_i - R$ is the distance of the boat to the circle $\mathcal{C}(p_c, R)$.
- q_i is the tack variable.

The distance d_{ij} between sailboat i and j can be evaluated as

$$d_{ij} = R\widehat{\mathrm{IJ}}, \tag{10}$$

where

$$\widehat{\text{IJ}} = \arcsin\left(s\sin\left(\alpha_i-\alpha_j\right)\right) \quad \text{If } s\sin\left(\alpha_i-\alpha_j\right)>0,$$
$$\widehat{\text{IJ}} = 2\pi-\arcsin\left(\sin\left(s\left(\alpha_i-\alpha_j\right)\right)\right) \quad \text{else.}$$

IV. Platooning control

To reach and maintain the desired distance d^*_{ij} between sailboats i and j, a control of sailboats velocity/acceleration is required. Since wind is the propulsion force for a sailboat, the sailboat's speed is proportional to the wind speed. A sailboat can not navigate inside the dead zone $\Omega_d(t)$ where it is upwind, i.e. against the wind direction. Thus, due to the unpredictable nature of the wind, the control of sailboat position/velocity is difficult or impossible in some scenarios. Theoretically, sailboat acceleration can be managed using the sail with a method like sail control exposed in [29]. However, maximal sailboat deceleration using the sail angle is very limited and it is not enough to perform a platooning.

Nevertheless, if the velocity/acceleration can not be tuned, a control of an average velocity/acceleration in a particular direction, $i.e.$ a projection of the velocity V_i/acceleration $\dot{V}_i$ on the reference path, is possible. To obtain this result, a solution is to make tacking inside a corridor, like in Filippov's continuation method [28].

In Section IV-A, the desired acceleration to reach a platooning is first described. Tack strategies to regulate sailboat velocity/acceleration using this desired acceleration are exposed in Section IV-B.

A. Desired acceleration

Since the behavior of the generic dynamics of a sailboat is complex and the values of its dynamic parameters are hard to obtain, a simplified model for the sailboat dynamics is considered to study the platooning problem:

$$M_i\dot{V}_i = u_i, \tag{11}$$

where M_i is the sailboat mass, u_i is the platooning control input. This simplification is feasible because the projected distance/acceleration d_{ij}/$\dot{V}_i$ on the axis AB or circle $\mathcal{C}(p_c, R)$ is considered here, and not the true distance/acceleration $\bar{d}_{ij}$/$\dot{v}_i$. Indeed, one can have $V_i = 0$ with $v_i = v_{\max}$ if $\theta_i = \theta^*_i + \frac{\pi}{2}$. Thus, the convergence of the heading θ_i in the general dynamics (1) to the desired heading $\bar{\theta}_i$ evaluated in Section IV-B guarantees the connection between (1) and (11), as shown in Appendix IX.

The projected distance d_{ij} is targeted to converge to the desired distance $d^*_{ij} = V_iT^*+d_{secu}$ as exposed in Section II-C. Thus the sailboat adjusts its dynamics following the control input

$$u_i = -k_{v,i}\left(V_i-V_j\right)+k_p\left(d_{ij}-d^*_{ij}\right), \tag{12}$$

where $k_{v,i} = \frac{M_i}{T^*}$ and $k_p > 0$ are the designed parameters. To obtain an optimal convergence and stability, it is advised to choose k_p close to $k_p = \frac{M}{2}$ where $M = \max_{i=1\ldots N}(M_i)$. Convergence of (11) using (12) is studied in Appendix VIII-A.

Remark: Only one neighbour is considered in this method to better manage the security distance and collision avoidance with the sailboat in the front.

Based on (11) and (12), one can define the desired acceleration of the sailboat $\dot{v}^*_i$:

$$\dot{V}^*_i = \frac{1}{M_i}\left(-k_{v,i}\left(V_i-V_j\right)+k_p\left(d_{ij}-d^*_{ij}\right)\right), \tag{13}$$

The desired acceleration $\dot{V}^*_i$ will be used in Section IV-B to control the sailboat heading, and so the projection of its velocity V_i.

Leader in the open-loop path: If sailboats follow a close-loop path, S_1 follows S_N. Else, as explained in Section˜II-C, S_1 follows a virtual leader of velocity v_0 and its control becomes

$$u_1 = -k_{v,i}\left(V_1-v_0\right). \tag{14}$$

B. Heading control

1) Main direction: To control the sailboat velocity, an usual method is to force the sailboat to make tack inside a corridor. By making tack, the boat moves at an average velocity V_i as required to perform the platooning. Figure 3 and Figure 4 show sailboat inside a corridor of width $2r_{secu}$. At the time instant t, if the desired acceleration $\dot{V}^*_i$ is negative, $i.e.$ sailboat i needs to slow down, it makes tacking following orientation $\bar{\theta}_i = \theta^*_i + q_i\gamma_i$, where $\gamma_i \in \left[0,\, \frac{\pi}{2}\right]$ and $q_i \in \{-1,\, 1\}$, thus $V_i = v_i\cos\left(\bar{\theta}_i-\theta^*_i\right) = v_i\cos\left(\gamma_i\right)$ if the reference path is a line, $V_i = v_i\cos\left(\gamma_i\right)\frac{R}{R_i}$ if the reference path is a circle. Variable γ_i is taken inside $\left[0, \frac{\pi}{2}\right]$ to avoid the sailboat to turn back, so $V_i \geq 0$. The tack variable q_i allows to alternate the tack when the sailboat reaches the limit of the corridor, as illustrated in Figure 3. As classic works on sailboats, a tacking strategy to move upwind is also defined, such $\bar{\theta}_i$ can not be chosen inside the dead zone $[\psi+\pi-\delta,\, \psi+\pi+\delta]$. The upwind tack strategy is to be described in the following subsection IV-B2.

2) Upwind tack strategy: If the course θ^*/$\bar{\theta}$ corresponds to a direction too close to the wind, a zigzag trajectory, namely upwind tack strategy, is used to move upwind while staying inside a corridor. This method defines the optimal heading to move upwind with the maximum velocity.

Contrary to classic tack strategies like in [10, 12, 23], here sailboats can not turn back to avoid collision with the sailboat behind. Thus, there exists case where only one hauled angle is possible between $\psi+\pi-\delta$ and $\psi+\pi+\delta$ for not turning back, $i.e.$ in area 1 and area 3 as exposed in Figure 4 in Section IV-B. Nevertheless, when the sailboat needs to slow down, it might conflict between the upwind tack strategy and the desired heading $\bar{\theta}_i$. Since the first priority is to avoid collision with other sailboats, the strategy to slow down will takes the priority over the upwind strategy.

To implement this strategy, the condition $c_0\left(i\right)$ is defined to be false if sailboat needs to slow down. While $c_0\left(i\right)$ is false, the upwind tack strategy will not be used. The detailed steps for the upwind tack strategy are described as Algorithm 1. This tack strategy is used to define the heading control.

3) Acceleration strategy for circular platooning: If sailboats are turning around a circular path, an approach to narrow the distance between sailboats faster can be proposed.

Algorithm 1 Upwind tack strategy

Require: $w_i \in \{-1, 1\}$ the upwind tack angle variable.

1: **if** $c_0(i)$ is true and the desired angle $\bar{\theta}_i$ is inside the dead zone, **then** define a transitional variable $\tilde{\theta}_i$ to indicate in which side of the corridor the sailboat desires to go:

$$\begin{aligned} &\text{If}\,(c_0(i) = \text{True})\,\&\,\left(\cos\left(\psi_{tw} - \bar{\theta}_i\right) + \cos(\delta) < 0\right) \\ &\quad \text{Put } \tilde{\theta}_i = \theta_i^* + q_i \frac{\pi}{2}. \end{aligned} \tag{15}$$

2: The closest hauled angle is chosen as the new desired angle $\bar{\theta}_i$:

3: **if** $\left(\cos\left(\psi_{tw} + \pi + \delta - \tilde{\theta}\right) < \cos\left(\psi_{tw} + \pi - \delta - \tilde{\theta}_i\right)\right)$ **then** $w_i = -1$

4: **else** $w_i = 1$

5: Take $\bar{\theta} = \psi_{tw} + \pi + w_i \delta$

6: **if** $\bar{\theta}_i$ indices sailboat has to go back **then** the other hauled angle is taken:

$$\begin{aligned} &\text{If } \cos\left(\theta_i^* - \bar{\theta}_i\right) < 0 \\ &\quad \bar{\theta}_i = \psi_{tw} + \pi - w_i \delta. \end{aligned} \tag{16}$$

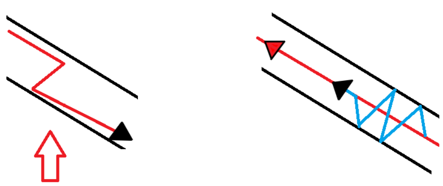


Figure 3: Left: tack to move front the wind (red arrow). Right: tack to slow down and take distance between the two sailboats.

For that, the corridor is cut into two corridors $\mathcal{C}_{int}$ and $\mathcal{C}_{ext}$ where sailboat i is inside $\mathcal{C}_{int}$ if $R_i \in [R_{int}, R]$ and sailboat i is inside $\mathcal{C}_{ext}$ if $R_i \in [R, R_{ext}]$. Since the angular velocity V_i is larger for the same velocity v_i when sailboat i is close to the center of the circle p_c, sailboat will move in $\mathcal{C}_{int}$ by taking the tangential trajectory of $\mathcal{C}(p_{int}, R_{int})$ if the sailboat needs to speed up while inside $\mathcal{C}_{ext}$. However, its strategy is not used if the sailboat is navigated upwind to avoid making tacks which slow down the sailboat during maneuver, *i.e.* it is never used when the sailboat is in Zone 4. Moreover, to obtain a smoother behavior, define $T_{\max}$ where $T_{\max} > T^*$ be the constant maximal time headway, where the acceleration strategy is used only if the time that the i-th sailboat takes to arrive at the position of its predecessor is larger than $T_{\max}$.

4) Heading control algorithm: The heading control is described by the Algorithm 2. The values of θ_i^*, e_i, V_i and q_i are the ones defined in Section III-A and Section III-B.

Note that this approach does not require the knowledge of sailboat dynamic parameters, thus it can be adapt to different models of sailboats.

V. Low level control

A. Rudder angle

In most work, see [10, 12], the rudder angle is computed using the heading θ. However, due to the action of the wind and the wave, the true course of the sailboat following

Algorithm 2 Heading control algorithm

Require: $\epsilon > 0$ design parameter, $\bar{s} = s$ if a circular path is follow, $\bar{s} = 1$ else.

$$\begin{aligned} c_1(i) = &\left(\cos(\pi - \delta) < \cos(\alpha_i - \psi_{tw}) < \cos(\delta)\right) \\ &\&\left(\sin(\alpha_i - \psi_{tw})\, s > 0\right) \end{aligned} \tag{17}$$

is the condition to indicate if sailboat i is inside the Zone 4.

1: **procedure**

2: Evaluate $\dot{V}_i^*$ using (13).

3: **if** $\dot{V}_i^* < 0$ **then** take $c_0(i) = \text{False}$ and the sail angle

$$\delta_s = -\text{sign}(\psi_{aw}) \min\left[|\pi - \psi_{aw}|, \delta_{s,\max}\right]. \tag{18}$$

4: **else** $c_0(i) = \text{True}$ and choose δ_s using (28).

5: **if** $|e_i| > r_{secu}$, the sailboat is outside the corridor. **then** take a heading to back inside:

$$q_i = \bar{s}\,\text{sign}(e_i), \tag{19}$$

$$\bar{\theta}_i = \theta_i^* + \frac{\pi}{2+\epsilon} q_i. \tag{20}$$

6: **else**

7: **if** $\dot{V}_i^* < 0$ sailboat i needs to slow down. **then** one take

$$\gamma_i = \min\left[\frac{\pi}{2}, \text{acos}\left[\text{sat}_1\left|\frac{\max\left(\dot{V}_i^*, 0\right)}{\max(\dot{v}_i, \epsilon)}\right|\right]\right], \tag{21}$$

$$\bar{\theta}_i = \theta_i^* + \gamma_i q_i. \tag{22}$$

8: **else** sailboat i needs to reduce the gap with its neigbhours. Consider the two cases:

9: **if** $V_i T_{\max} > d_{ij} - d_{secu}$ or the reference path is a line **then** take the default path $\bar{\theta}_i = \theta_i^*$.

10: **else**, i.e. if $V_i T_{\max} < d_{ij} - d_{secu}$ and the reference path is a circle, sailboat if far from its target. Thus, acceleration strategy for circular platooning is used

$$\begin{aligned} &\text{If } \left(V_i < \frac{d_{ij} - d_{secu}}{T_{\max}}\right) \& \left(R_i > R - 0.8 r_{secu}\right) \\ &\quad \& \left(c_1(i) = \text{False}\right): \\ &\quad \gamma_i = \text{acos}\left(\frac{R_{int}}{R_i}\right) \quad q_i = s, \end{aligned} \tag{23}$$

$$\bar{\theta}_i = \theta_i^* + \gamma_i q_i, \tag{24}$$

$$\text{Else } \bar{\theta}_i = \theta_i^*. \tag{25}$$

11: Upwind tack strategy described in Section IV-B2 is employed.

ϕ is often different to its theoretical course following the heading θ, see [26]. This discrepancy is mainly observed when sailing close-hauled, where wind drifts the sailboat from a line it follows. Thus, combining ideas from [26] and [12], the following rudder control uses the course angle to compensate the perturbations:

$$\delta_r = \delta_{r,\max} \sin\left(\Theta - \bar{\theta}\right) \quad \text{if } \cos\left(\Theta - \bar{\theta}\right) \geq 0, \tag{26}$$

$$\delta_r = \delta_{r,\max} \text{sign}\left(\sin\left(\Theta - \bar{\theta}\right)\right) \quad \text{else}. \tag{27}$$

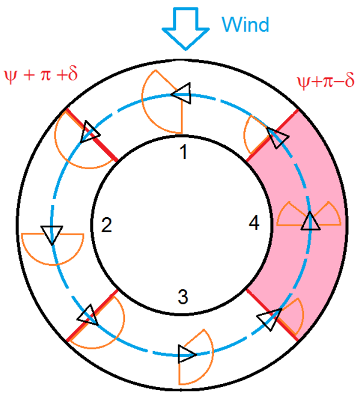


Figure 4: Platooning circle. In orange, the possible heading if sailboat must not to go upwind and not go back. Zone 1,2,3 and 4 are based on the hauled angle $\psi + \pi - \delta$ and $\psi + \pi + \delta$. Zone 4 (in pink) is the area with the least possibility to manoeuvre.

where $\bar{\theta}$ is the desired heading, $\Theta = \phi$ if $\cos(\theta - \phi) - \cos(\epsilon_\theta) \geq 0$ with $\epsilon_\theta \in \left[0, \frac{\pi}{2}\right]$ as the design parameter angle, $\Theta = \theta$ else.

B. Sail angle

Sailboats are vehicles which require a considerable velocity to be controlled, that is why usual techniques tune sails so as to obtaining the optimal velocity and acceleration. An optimal sail angle based on the sailing diagram presented in [12, 10] is proposed. The optimal sail angle is modified to be independent of the desired heading $\bar{\theta}$ to be more stable. It can be defined as

$$\delta_s^{opt} = \frac{\pi}{2}\left(\frac{\cos(\psi_{aw}) + 1}{2}\right). \tag{28}$$

Since the sail cannot hold against the wind, the sail opening δ_s cannot exceed a certain angle. This condition can be expressed as

$$\delta_s \in -\text{sign}(\psi_{aw}) * [0, \delta_{s,M}], \tag{29}$$

where $\delta_{s,M} = \min(|\pi - |\psi_{aw}||, \delta_{s,\max})$, with $\delta_{s,\max}$ the maximal sail opening defined by the physical constraint.

Thus, using δ_s^{opt}, $\delta_{s,\max}$ and $\delta_{s,M}$, one takes if $\dot{V}_i^* > 0$

$$\delta_s = -\text{sign}(\psi_{aw}) \min\left[\left|\delta_s^{opt}\right|, \delta_{s,M}, \right], \tag{30}$$

and takes if $\dot{V}_i^* < 0$

$$\delta_s = -\text{sign}(\psi_{aw})\, \delta_{s,M}. \tag{31}$$

This technique is simple to implement with a smaller calculation time. Moreover, this strategy does not require knowledge of the dynamic parameters to be implemented. However, this technique can not allow to control the sailboat velocity.

VI. Simulations

A. Simulated dynamic model

Based on [18], sailboats are represented in the simulations by the following non-linear differential state equations:

$$\dot{x} = v\cos(\theta) - w\sin(\theta), \tag{32}$$
$$\dot{y} = v\sin(\theta) + w\cos(\theta), \tag{33}$$
$$\dot{\theta} = \omega, \tag{34}$$
$$p_9\dot{v} = g_s\sin(\delta_s) - g_{rv}p_{11}\sin(\delta_r) - p_2 v|v| + p_1 a_{tw}^2\cos(\psi_{tw} - \theta), \tag{35}$$
$$p_9\dot{w} = -g_{ru}p_{11}\cos(\delta_r) - p_2 w|w| + p_1 a_{tw}^2\sin(\psi_{tw} - \theta), \tag{36}$$
$$p_{10}\dot{\omega} = g_s(p_6 - p_7\cos(\delta_s)) - g_{rv}p_8\cos(\delta_r) - p_3\omega v, \tag{37}$$

where g_s, g_{rv} and g_{ru} are forces on the sail and the rudder

$$g_{rv} = p_5 v^2 \sin(\delta_r), \tag{38}$$
$$g_{rw} = p_5 w|w|\cos(\delta_r), \tag{39}$$
$$g_s = p_4 a_{aw}\sin(\delta_s - \psi_{aw}), \tag{40}$$

where a_{aw}, ψ_{aw} and a_{tw}, ψ_{tw} are the apparent wind and the true wind described in section II-B. Terms $p_1 a_{tw}^2$ represents the wind force on the hull. Consider here the keel compensates forces of the sail on u. Tangential and the angular friction forces are described by $p_2 v^2$ and $p_3\omega v$, respectively. All parameters $p_{i,1}$, i.e paramters $p_{i,1}$ associated to sailboat 1 can be found in Table II. Parameters $p_{i,j}$ associated to other sailboat j $j \in \mathcal{N}$ are randomly generated using $p_{i,1}$ such $p_{i,j} = (1 + 0.1randn(1))p_{i,1}$. Remark since parameters $p_1, \ldots, p_{11}$ are difficult to obtain, the strategies proposed in previous sections do not require them.

B. Parameters

To evaluate performance of the proposed algorithms, simulations are performed. Consider $N = 8$ identical sailboats, where parameters of the simulated sailboat are expressed in Table II, $M_i = p_9$, $C_i = p_2$ and $v_{\max} = 5$. Take $d_{secu} = \frac{M_i}{C_i}v_{\max} + p_7 + p_8$, $r_{secu} = \frac{M_i}{C_i}v_{\max}$. Take the maximal angle $\delta = \frac{\pi}{4}$, $\delta_{r,\max} = \frac{\pi}{4}$ and $\delta_{s,\max} = \frac{\pi}{2}$. Take a duration of $T = 250$ s, T^* is to be specified in each simulation. The wind direction and strength are $\psi_{tw} = \frac{\pi}{2} + \frac{\pi}{2}\sin(\frac{\pi}{2}t/50)$ and $a_{tw} = 10 + 2\sin(\frac{\pi}{2}t/50)$. The discrepancy between the desired distance d_{ij}^* and the current distance d_{ij} between sailboats i and j is expressed in Figure 5 and Figure 6 such

$$D = 100\left|d_{ij} - d_{ij}^*\right|/d_{ij}^*. \tag{41}$$

When $D = 0$, the headway time T^* or the security distance d_{secu} is respected between sailboats i and j.

Two cases are compared in this section for the circular path:

Case 1: the optimal receding horizon control (RHC) algorithm studied in [13] for nonlinear vehicles. The algorithm is used to evaluate $u_i = [\delta_{s,i}, \delta_{r,i}]$ such sailboats converge to the platooning position and velocity. The dynamics model described by (32)-(37) is used inside the optimization loop. As method is developed for 1-dimensional space, projection of p_i, v_i, $\dot{v}_i$ on circular path is considered to perform it. Constraints $u_i \in [[-\frac{\pi}{2}, \frac{\pi}{2}] \times [-\frac{\pi}{4}, \frac{\pi}{4}]]$ and P_i inside the corridor are taken.

Case 2: heading control platooning described in Section 4.2 and the tack strategy in Section 3.1 is used.

	parameter	value		parameter	value		parameter	value
p_1	drift coefficient	0.03	$p_5[\text{kgs}^{-1}]$	rudder lift	1500	$p_9[\text{kg}]$	mass of boat	300
$p_2[\text{kgs}^{-1}]$	tangential friction	40	$p_6[\text{m}]$	distance to sail	0.5	$p_{10}[\text{kgm}^2]$	moment of inertia	400
$p_3[\text{kgm}]$	angular friction	6000	$p_7[\text{m}]$	distance to mast	0.5	$p_{11}-$	rudder break coefficient	0.2
$p_4[\text{kgs}^{-1}]$	sail lift	200	$p_8[\text{m}]$	distance to rudder	2			

Table II: Model parameters value, from [18]

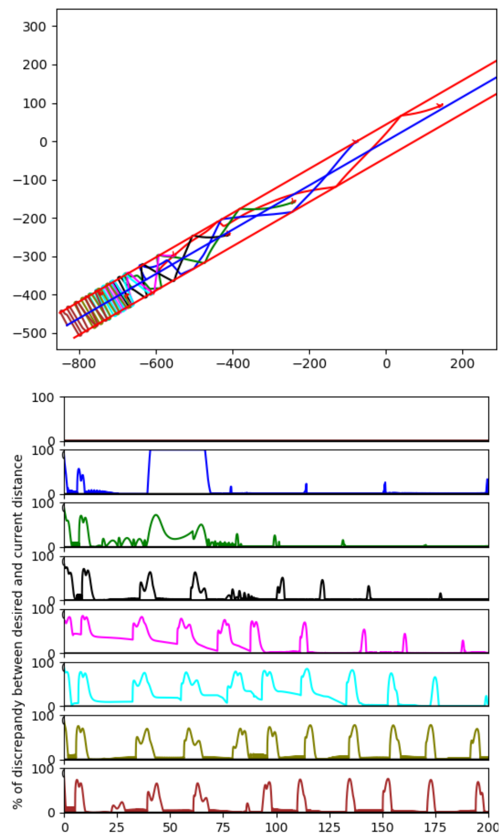


Figure 5: Platooning with a linear path.. sailboat's $1, 2, ..., 8$ trajectories are colors {red, blue, green, black, magenta, cyan, khaki, brown}. $T^* = 40$.

C. Simulation with linear path

Platooning with a linear path is simulated using $A = [-433, -250]^T$, $B = [433, 250]^T$, so $\widehat{AB} = \frac{\pi}{6}$. Initial positions of sailboats are $P_i = (500 - d_{secu}(i-1)) * [-\cos(AB), -\sin(AB)]$.

The sailboat trajectories can be observed in Figure 6. Sailboats make a large number of tacks at the beginning of the simulation to slow down: they are waiting the sailboat in the front to gain distance before being able to speed up. One by one, they leave the initial pack while maintaining the desired distance between them. In Figure 5, since a perfect control of sailboat velocity/position cannot be obtained as explained in Section IV, the error is oscillating around zero and not always converge. However, one can observer $d_{ij} - d^*_{ij}$ is close to zero in most of the time, thus the headway time T^* and/or the security is respected among all sailboats.

D. Simulation with circular path

Platooning with a circular path is simulated using $R_{int} = \frac{N}{2\pi}(\max(d_{secu}, T^*) + p_7 + p_8)$, $R = R_{int} + r_{secu}$, $R_{ext} = R + r_{secu}$ and $p_c = [0, 0]^T$. Initial positions of sailboats are $P_i = (R+5) * \left[\cos\left(\frac{2\pi}{N}i\right), \sin\left(\frac{2\pi}{N}i\right)\right]$. Rotation direction is $s = 1$.

Results of Case 1 are presented in Figure 6(a) and results of Case 2 are presented in Figure 6(b). In Figure 6(a), the nearly circular trajectories performed by sailboats show the RHC method uses sail opening to regulate sailboats velocity and to maintain the desired distance between boats. However, several points make this method difficult to implement in practice. First, this method requires the knowledge of the dynamic model of the vehicle, perfectly known here but difficult to obtain in real cases. Second, the control of the sail opening requires several seconds on a real boat, where it is supposed to be instantaneous here, making it impossible to obtain a control of sailboat velocity using sail as accurate as in the simulation. Finally, the optimization process used in this method requires a long processing time, which makes it difficult to implement without a performer processor in embedded system.

In Figure 6(b), a large number of tacks in sailboats trajectories is observed to regulate boats velocity. Distance errors are close to ones obtain using the RHC optimization. However, this method does not require knowledge of sailboat dynamics with a much lower processing time, making it simpler to implement than optimization methods like in Case 1.

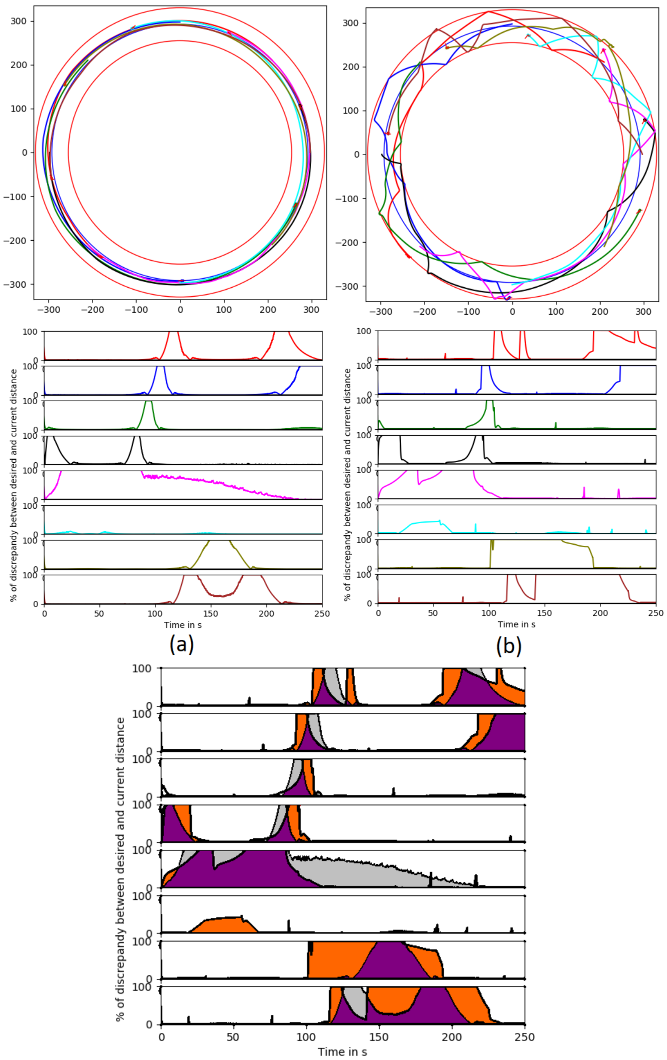
VII. Conclusion

In this paper, a platooning control with a constant time headway for sailboats has been defined. Problems of platooning following a linear and a circular path for sailboats are also studied. For that, a desired acceleration of the sailboat to obtain a platooning is exposed, and a tack strategy to regulate sailboat velocity has been described to follow its desired acceleration. Proof for the convergence of the desired distance between sailboats has been provided. A second tack strategy to move upwind has been described, and its compatibility with the first strategy has been studied. Simulation results demonstrate the effectiveness of the approach.

In future work, this method will be implemented on a real autonomous sailboat fleet to test its effectiveness. Moreover, a more robust control will be designed for the unspecific sailboat parameters, and the management of communications, like reduce number of exchange using event-triggered approach, will also be considered.

Acknowledgment

This work was supported by the Engineering and Physical Sciences Research Council (EPSRC) of the U.K. under Grant EP/R005532/1.



(c)

Figure 6: Platooning with a circular path. (a) RHC optimization algorithm. (b) heading control. (c) comparison of distance errors: the platooning with circular path's error in orange, RHC optimization's error in grey and shared error in purple. Desired headway time $T^* = 40$. Colors line are sailboat trajectories in top, and corresponding color in bottom are associated performance. sailboat's $1, 2, ..., 8$ trajectories are colors {red, blue, green, black, magenta, cyan, khaki, brown}.

VIII. Appendix

A. Proof of convergence

Two cases are considered at first. If sailboats follow a close-loop path, S_1 follows S_N. If sailboats follow an open path, S_1 is defined as leader and its control becomes

$$M_i \dot{V}_i = -k_{v,i} \left(V_i - v_0\right), \tag{42}$$

where (42) can be solved to obtain $V_i(t) = V_i(0) e^{-\frac{M_i}{k_{v,i}} t} + \left(1 - e^{-\frac{M_i}{k_{v,i}} t}\right) v_0$. One deduce the sailboat 1 velocity converges to the desired velocity v_0. Consider now the followers : let define $I = 1$ if sailboats follow a close-loop path or without leader, $I = 2$ if sailboats follow S_1 as a leader.

Consider now the others sailboats. A Lyapunov function $\mathbb{V} = \frac{1}{2} \sum_{i=1}^{N} M_i \left(d_{ij} - d^*_{ij}\right)^2$ is defined. The derivative of $\mathbb{V}$ is

$$\dot{\mathbb{V}} = \sum_{i=I}^{N} M_i \left(d_{ij} - d^*_{ij}\right) \left(\dot{d}_{ij} - \dot{d}^*_{ij}\right), \tag{43}$$

where $\dot{d}_{ij} = V_j - V_i$ and $\dot{d}^*_{ij} = \dot{V}_i T^*$, thus $\dot{\mathbb{V}} = \left(d_{ij} - d^*_{ij}\right) \left(M_i \left(V_j - V_i\right) - M_i \dot{V}_i T^*\right)$. Using (12), one gets

$$\begin{aligned}\dot{\mathbb{V}} = &\sum_{i=I}^{N} \left(d_{ij} - d^*_{ij}\right) \left(M_i \left(V_j - V_i\right) + T^* k_{v,i} \left(V_i - V_j\right)\right. \\ &\left. -T^* k_p \left(d_{ij} - d^*_{ij}\right)\right) \qquad (44)\\ = &-T^* k_p \sum_{i=1}^{N} \left(d_{ij} - d^*_{ij}\right)^2 + \sum_{i=1}^{N} \left(d_{ij} - d^*_{ij}\right) \left(V_j - V_i\right) \\ &\left(M_i - T^* k_{v,i}\right), \qquad (45)\end{aligned}$$

and since $k_{v,i} = \frac{M_i}{T^*}$, one has $\dot{\mathbb{V}} = -T^* k_p \sum_{i=I}^{N} \left(d_{ij} - d^*_{ij}\right)^2$, thus $\dot{\mathbb{V}} \leq -T^* k_p \sum_{i=I}^{N} \frac{M_i}{M} \left(d_{ij} - d^*_{ij}\right)^2$ where $M = \max_{i=1...N}(M_i)$ and so

$$\dot{\mathbb{V}} \leq -\frac{2T^* k_p}{M} \mathbb{V}. \tag{46}$$

which is always negative. Define the function $\mathbb{U}$ such $\dot{\mathbb{U}} = -\left(\frac{2T^* k_p}{M}\right) \mathbb{U}$ and $\mathbb{U}(0) = \mathbb{V}(0)$. One gets $\mathbb{U}(t) = \mathbb{V}(0) e^{-\left(\frac{2T^* k_p}{M}\right) t}$. Using the comparison theorem in [11], one obtains $\mathbb{V}(t) \leq \mathbb{V}(0) e^{-\frac{2T^* k_p}{M} t}$. Thus, $\mathbb{V} \geq 0$, $\dot{\mathbb{V}} \leq 0$ and $\mathbb{V}$ converges to zero. Using the Lyapunov theorem, $d_{ij}(t)$ converges to d^*_{ij} when $t \to \infty$ if the desired values $\dot{V}^*_i$ are respected by sailboats. Note M_i must be known exactly by Agent i for guaranteeing the stability of the sailboat while M can be chosen such be larger than the sailboat mass so as to guarantee the stability.

To synchronize leader and follows convergence, (42) and (46) need to converge at the same velocity. Thus, to have $\frac{M_i}{k_{v,i}} = \frac{2T^* k_p}{M}$ and since $k_{v,i} = \frac{M_i}{T^*}$, one gets $k_p = \frac{M}{2}$. Thus, it is advised to choose $k_p = \frac{M}{2}$ to obtain an optimal convergence and stability.

B. Turning distance

$d^{(1)}_{secu}$ is to be studied. For the sailboat i may turn to avoid the sailboat j in the front, l_i, L_j, and $r_{r,i}$ must be considered. If sailboat moves out of the corridor while it tries to avoid its predecessor, or to move upwind, the sailboat must be able to perform a minimum of two tacks. Thus, it will perform three rotations of $\frac{\pi}{2}$. Using Dubins approach [8], it requires a distance of $3r_{r,i}$. Finally, since sailboats inside the fleet are heterogeneous, the bigger turning angle $r_{r,i}$ must be considered. Thus, let define the first condition $d^{(1)}_{secu}$ on d_{secu} such

$$d^{(1)}_{secu} = 3 \max_{i=1:N} \left(r_{r,i}\right) + \max_{i=1:N} \left(l_i\right) + \max_{i=1:N} \left(L_i\right). \tag{47}$$

C. Stopping distance

As exposed in (1) the dynamics of a sailboat can be summarized as $M_i \dot{v} = \tau_f + \tau_r + \tau_s + \tau_w$. Since a sailboat can stop only when the wind is not behind, consider the sailboat in this configuration, so $\tau_w \leq 0$. When the sailboat tries to slow down, let $\delta_s = \pi - \psi_{aw}$ as chosen in (18) in Section IV-B such that sail cannot help the sailboat to move forward, so $\tau_s \leq 0$. Finally, action of rudder is always in opposition to the movement of the sailboat, thus $\tau_r \leq 0$. Then, to evaluate the stopping distance, considering the worst-case dynamics of the sailboat

$$M_i \dot{v} = \tau_f. \tag{48}$$

The friction force τ_f is to be described as follows. When the sailboat velocity is low ($v < 5$m/s), one has $\tau_f^{low} = -C_i v$ where C_i is the friction coefficient, a function of the shape of the sailboat and the fluid density. Else, when the sailboat velocity is large, one has $\tau_f^{large} = -C_i v^2$. Again, since $\left|\tau_f^{large}\right| > \left|\tau_f^{low}\right|$ when the velocity is large and the worse case for the stopping distance is considered, take $\tau_f = \tau_f^{low}$. Thus

$$M_i \dot{v} = -C_i v \Leftrightarrow \dot{v} = -\frac{C_i}{M_i} v. \tag{49}$$

Solve (49), one has $v = \beta e^{-\frac{k_{x,i}}{M_i} t}$ with β a constant. Using initial condition, one gets

$$v(t) = v(0)\, e^{-\frac{C_i}{M_i} t}. \tag{50}$$

By integrating (50), one obtains the stopping

$$d(t) = -\frac{M_i}{C_i} v(0)\, e^{-\frac{C_i}{M_i} t} + \gamma. \tag{51}$$

Using initial condition $d(0) = 0$, one has $\gamma = \frac{M_i}{C_i} v(0)$, so

$$d(t) = \frac{M_i}{C_i} v(0) \left(1 - e^{-\frac{C_i}{M_i} t}\right), \tag{52}$$

which converges to $d(t) = \frac{M_i}{C_i} v(0)$ when $t \to \infty$. Taking $v(0) = v_{\max}$ and adding the length of the sailboat to avoid collision, one gets $d_{secu,i} = \frac{M_i}{C_i} v_{\max} + l_i + L_i$ for a sailboat i. d_{secu} is the biggest distance $d_{secu,i}$ between all sailboats in the fleet.

IX. Proof of Stability and Convergence of Control Orientation

In this Appendix, the orientation θ_i is to converge to the desired orientation $\bar{\theta}$.

A. Sailboat dynamics model

As exposed in (2), a general nonlinear dynamic model of a sailboat rotation can be expressed as $J\ddot{\phi} = C_r + C_s + C_k + C_h$. In particular, one may write $C_r \approx -\beta v^2 \sin(2\delta_r)$ where δ_r is the rudder angle, v the sailboat velocity, and $\beta > 0$ a constant based on sailboat shape, water density, lift and drag coefficients. Knowledge of J, C_s, C_k, C_h and β are not required for implementing the following proof.

B. Proof of stability

Define $\omega = \dot{\phi}$. The desired rotation velocity $\bar{\omega}$ and desired rotation acceleration $\dot{\bar{\omega}}$ are chosen such $\bar{\omega} = \omega - \sin\left(\Theta - \bar{\theta}\right)$ and $\dot{\bar{\omega}} = 0$. Thus, one can deduce

$$\omega - \bar{\omega} = \sin\left(\Theta - \bar{\theta}\right). \tag{53}$$

Note $\dot{\bar{\omega}}$ tries to make $\dot{\omega}$ to converge to zero and $\bar{\omega}$ tries to make ω to converge to zero when the sailboat reaches the desired orientation.

Define the Lyapunov function V such

$$V = \frac{1}{2} J (\omega - \bar{\omega})^2 \left(1 - \frac{1}{1 + \ln\left(2 - \cos\left(\Theta - \bar{\theta}\right)\right)}\right), \tag{54}$$

and put $W = 1 - \frac{1}{1+\ln\left(2-\cos\left(\Theta-\bar{\theta}\right)\right)}$ such $W \geq 0$ and $W = 0$ when $\Theta - \bar{\theta} = 0$. Remark V has been chosen such $V = 0$ when $\Theta = \bar{\theta}$ and decreases when Θ converges to $\bar{\theta}$.

The derivative of V is

$$\dot{V} = (\omega - \bar{\omega}) J (\dot{\omega} - \dot{\bar{\omega}}) W + \frac{1}{2} J (\omega - \bar{\omega})^2 dW, \tag{55}$$

with

$$dW = -\frac{-\left(\frac{\sin\left(\Theta-\bar{\theta}\right)(\omega-\bar{\omega})}{2-\cos\left(\Theta-\bar{\theta}\right)}\right)}{\left(1 + \ln\left(2 - \cos\left(\Theta - \bar{\theta}\right)\right)\right)^2} \tag{56}$$

$$= -\frac{\sin\left(\Theta - \bar{\theta}\right)(\omega - \bar{\omega})}{\left(2 - \cos\left(\Theta - \bar{\theta}\right)\right)\left(1 + \ln\left(2 - \cos\left(\Theta - \bar{\theta}\right)\right)\right)^2}. \tag{57}$$

Using (53), one has

$$dW = -\frac{\sin\left(\Theta - \bar{\theta}\right)^2}{\left(2 - \cos\left(\Theta - \bar{\theta}\right)\right)\left(1 + \ln\left(2 - \cos\left(\Theta - \bar{\theta}\right)\right)\right)^2} \leq 0 \tag{58}$$

Thus

$$\dot{V} \leq (\omega - \bar{\omega}) J \ddot{\phi} W. \tag{59}$$

Using (2), (53) and put $S_\tau = C_s + C_k + C_h$, one gets

$$\dot{V} = \sin\left(\Theta - \bar{\theta}\right)(C_r + S_\tau) W \tag{60}$$

Since the sailor can only use the rudder to control sailboat orientation and rudder action is restricted by $|C_r| \leq \beta v^2 \sin(\delta_{r,\max})$. If $|C_r| \leq |S_\tau|$ with $\mathrm{sgn}(C_r) \neq \mathrm{sgn}(S_\tau)$, then the sailboat orientation is uncontrollable. Then, suppose we are in a case where the sailboat is controllable, so $\mathrm{sgn}(\tau_r) = \mathrm{sgn}(S_r)$ or $|C_r| > |S_\tau|$. Thus, one may write $C_r + S_\tau = \alpha \mathrm{sgn}(\tau_r)$ where $\alpha > 0$ (case $\alpha = 0$ induce $C_r + S_\tau = 0$, so sailboat is uncontrollable too. Note if $C_r + S_\tau = 0$, one has $\dot{V} = \frac{1}{2} J (\omega - \bar{\omega})^2 dW \leq 0$). Define also $\alpha_{\min}$ the smallest value of α such $C_r + S_\tau = \alpha \mathrm{sgn}(C_r)$ with $\mathrm{sgn}(C_r) \neq \mathrm{sgn}(S_\tau)$, *i.e.* $\alpha_{\min} = \min(\alpha)\ \forall (C_r, S_\tau) \neq \{0, 0\}$ and $\mathrm{sgn}(C_r) \neq \mathrm{sgn}(S_\tau)$. One gets

$$\begin{aligned} \dot{V} &\leq \sin\left(\Theta - \bar{\theta}\right) \alpha \mathrm{sgn}(C_r) W \\ &\leq \sin\left(\Theta - \bar{\theta}\right) \alpha \mathrm{sgn}\left(-\beta v^2 \sin(2\delta_r)\right) W \\ &\leq -\alpha \sin\left(\Theta - \bar{\theta}\right) \mathrm{sgn}\left(\sin(\delta_r)\cos(\delta_r)\right) W. \end{aligned} \tag{61}$$

Remind $\delta_r \in [-\delta_{r,\max}, \delta_{r,\max}]$ where $0 < \delta_{r,\max} < \frac{\pi}{2}$, thus $\cos(\delta_r) \geq 0$. Put $S = \left|\sin\left(\Theta - \bar{\theta}\right)\right|$ if $\cos\left(\Theta - \bar{\theta}\right) \geq 0$ and $S = 1$ else. Using (26)-(27), one gets

$$\begin{aligned} \dot{V} &\leq -\alpha \sin\left(\Theta - \bar{\theta}\right) \mathrm{sgn}\left(\sin\left(\delta_{r,\max} S \mathrm{sgn}\left(\sin\left(\Theta - \bar{\theta}\right)\right)\right)\right) W \\ &\leq -\alpha \sin\left(\Theta - \bar{\theta}\right) \mathrm{sgn}\left(\sin\left(\Theta - \bar{\theta}\right)\right) W \\ &\leq -\alpha \left|\sin\left(\Theta - \bar{\theta}\right)\right| W \leq 0. \end{aligned} \tag{62}$$

Thus, $V \geq 0$ and $\dot{V} \leq 0$. Using the Lyapunov theorem, one deduces the rudder control (26)-(27) allows a stable control in orientation and rotation velocity if the sailboat orientation is controllable by the rudder.

C. Proof of convergence

Let now show the convergence of V. Using (62), one has

$$\dot{V} \leq -\left|\sin\left(\Theta - \bar{\theta}\right)\right|\left(\frac{2\alpha}{J}\right)\left(\frac{1}{2}JW\frac{\left(\omega-\bar{\omega}\right)^2}{\left(\omega-\bar{\omega}\right)^2}\right), \quad (63)$$

and since $\omega - \bar{\omega} = \sin\left(\Theta - \bar{\theta}\right)$, one has

$$\begin{aligned}\dot{V} &\leq -\left|\sin\left(\Theta - \bar{\theta}\right)\right|\left(\frac{2\alpha}{J}\right)\left(\frac{1}{2}JW\frac{\left(\omega-\bar{\omega}\right)^2}{\sin\left(\Theta-\bar{\theta}\right)^2}\right)\\ &\leq -\left(\frac{2\alpha}{J}\right)\left(\frac{1}{2}JW\frac{\left(\omega-\bar{\omega}\right)^2}{\left|\sin\left(\Theta-\bar{\theta}\right)\right|}\right) \leq -\left(\frac{2\alpha_{\min}}{J}\right)V.\end{aligned} \quad (64)$$

Let define the function U such $\dot{U} = -\left(\frac{2\alpha_{\min}}{J}\right)U$ and $U(0) = V(0)$. One shows than $U(t) = V(0)\,e^{-\left(\frac{2\alpha_{\min}}{J}\right)t}$. Using the comparison theorem in [11], one has $V(t) \leq U(t)$, so

$$V(t) \leq V(0)\,e^{-\left(\frac{2\alpha_{\min}}{J}\right)t}. \quad (65)$$

From (65), one may deduce $\lim_{t\to\infty} V(t) = \lim_{t\to\infty} V(0)\,e^{-\left(\frac{2\alpha_{\min}}{J}\right)t} = 0$. So, $V(t)$ converges to zero. Since $V = \frac{1}{2}J\left(\omega-\bar{\omega}\right)^2\left(1-\frac{1}{1+\ln\left(2-\cos\left(\Theta-\bar{\theta}\right)\right)}\right)$, $V(t) = 0$ induce (a) $(\omega - \bar{\omega}) = 0$ or (b) $\left(1-\frac{1}{1+\ln\left(2-\cos\left(\Theta-\bar{\theta}\right)\right)}\right) = 0$. Let study the two solutions. If (b):

$$\begin{aligned}1-\frac{1}{1+\ln\left(2-\cos\left(\Theta-\bar{\theta}\right)\right)} &= 0\\ 2-\cos\left(\Theta-\bar{\theta}\right) &= 1 \Leftrightarrow \Theta = \bar{\theta}.\end{aligned} \quad (66)$$

If (b): $(\omega - \bar{\omega}) = 0$, one has $(\omega - \bar{\omega}) = 0 \Leftrightarrow \sin\left(\Theta - \bar{\theta}\right) = 0 \Leftrightarrow \Theta = \bar{\theta}$. In both cases, one has $\Theta = \bar{\theta}$, which indicates that the rudder control (26)-(27) allows Θ to converge to the desired value $\bar{\theta}$.

X. Proof of convergence sailboats to platooning

In Appendix VIII-A, one shows that global system converges to a stable platooning if the desired value $\dot{V}_i^*$ is respected. Since $\dot{V}_i = \dot{v}_i\cos(\bar{\theta}_i - \theta_i^*) = \dot{v}_i\cos(q_i\gamma_i)$ and (21), one has $\dot{V}_i = \dot{V}_i^*$ if θ_i has converged to $\bar{\theta}$.

In Appendix IX, one shows that rudder control allows θ_i to converge to $\bar{\theta}$.

Combining these two proofs, since $\dot{V}_i$ guarantees a stable convergence to the platooning and rudder guarantee a stable convergence from $\dot{V}_i$ to $\dot{V}_i^*$, that the control is stable and the platooning tracking error converges to zero.

Note that in order to avoid disturbance propagation inside the platooning, the convergence of the control of direction must be faster than the platooning convergence studied in Appendix VIII-B. Thus, (46) is supposed to decrease slower than (65), which induces $\frac{2\alpha_{\min}}{J_i} \geq \frac{2T^*k_p}{M_i}$. Studying it, one obtains the condition $k_p \leq \frac{M_i\alpha_{\min}}{T^*J_i}$. However, since $\alpha_{\min}$ is not simple to evaluate or measure, condition on k_p cannot be used easily and a further study of this problem will be included in future work.

References

[1] Abualhoul, M., Marouf, M., Shagdar, O., and Nashashibi, F. (2013). Platooning control using visible light communications: A feasibility study. In *IEEE Intelligent Transportation Systems-(ITSC)*, pages 1535–1540.

[2] Bernardo, M., Falcone, P., Salvi, A., and Santini, S. (2016). Design, analysis, and experimental validation of a distributed protocol for platooning in the presence of time-varying heterogeneous delays. *IEEE Transactions on Control Systems Technology*, 24(2):413–427.

[3] Cabrera-Gámez, J., Isern-González, J., Hernández-Sosa, D., Domínguez-Brito, A. C., and Fernández-Perdomo, E. (2013). Optimization-based weather routing for sailboats. In *Robotic Sailing 2012*, pages 23–33.

[4] Chehardoli, H. and Ghasemi, A. (2018). Adaptive centralized/decentralized control and identification of 1-d heterogeneous vehicular platoons based on constant time headway policy. *IEEE Transactions on Intelligent Transportation Systems*, pages 1–11.

[5] Delimpaltadakis, I. M., Bechlioulis, C. P., and Kyriakopoulos, K. J. (2018). Decentralized platooning with obstacle avoidance for car-like vehicles with limited sensing. *IEEE Robotics and Automation Letters*, 3(2):835–840.

[6] di Bernardo, M., Salvi, A., and Santini, S. (2015). Distributed consensus strategy for platooning of vehicles in the presence of time-varying heterogeneous communication delays. *IEEE Transactions on Intelligent Transportation Systems*, 16(1):102–112.

[7] Dolk, V., Ploeg, J., and Heemels, H. (2017). Event-triggered control for string-stable vehicle platooning. *IEEE Transactions on Intelligent Transportation Systems*, 18(12):3486–3500.

[8] Dubins, L. (1957). On curves of minimal length with a constraint on average curvature, and with prescribed initial and terminal positions and tangents. *American Journal of mathematics*, 79(3):497–516.

[9] Guanetti, J., Kim, Y., and Borrelli, F. (2018). Control of connected and automated vehicles: State of the art and future challenges. *Annual Reviews in Control*.

[10] Jaulin, L. and Le Bars, F. (2013). A simple controller for line following of sailboats. In *Robotic Sailing 2012*, pages 117–129.

[11] Khalil, H. K. (2002). *Nonlinear Systems, 3rd ed.* Upper Saddle River,NJ: Prentice-Hall.

[12] Le Bars, F. and Jaulin, L. (2013). An experimental validation of a robust controller with the vaimos autonomous sailboat. In *Robotic Sailing 2012*, pages 73–84.

[13] Li, H., Shi, Y., and Yan, W. (2016). Distributed receding horizon control of constrained nonlinear vehicle formations with guaranteed γ-gain stability. *Automatica*, 68:148–154.

[14] Li, S., Zheng, Y., Li, K., Wu, Y., Hedrick, J. K., Gao, F., and Zhang, H. (2017). Dynamical modeling and distributed control of connected and automated vehicles: Challenges and opportunities. *IEEE Intelligent Transportation Systems Magazine*, 9(3):46–58.

[15] Linsenmayer, S. and Dimarogonas, D. (2015). Event-triggered control for vehicle platooning. In *Proc IEEE ACC*, pages 3101–3106.

[16] Liu, B., Jia, D., Lu, K., Ngoduy, D., Wang, J., and Wu, L. (2017). A joint control-communication design for reliable vehicle platooning in hybrid traffic. *IEEE Transactions on Vehicular Technology*.

[17] Medina, A., van de Wouw, N., and Nijmeijer, H. (2017). Cooperative intersection control based on virtual platooning. *IEEE Transactions on Intelligent Transportation Systems*.

[18] Melin, J. (2015). Modeling, control and state-estimation for an autonomous sailboat.

[19] Melin, J., Dahl, K., and Waller, M. (2016). Modeling and control for an autonomous sailboat: a case study. In *Robotic Sailing 2015*, pages 137–149.

[20] Peng, Z. and Wang, J. (2018). Output-feedback path-following control of autonomous underwater vehicles based on an extended state observer and projection neural networks. *In Proc. IEEE Trans. on Syst., Man, and Cyber.: Systems*, 48(4):535–544.

[21] Peng, Z., Wang, J., and Wang, D. (2017). Distributed containment maneuvering of multiple marine vessels via neurodynamics-based output feedback. *IEEE Transactions on Industrial Electronics*, 64(5):3831–3839.

[22] Peng, Z., Wang, J., and Wang, D. (2018). Distributed maneuvering of autonomous surface vehicles based on neurodynamic optimization and fuzzy approximation. *In Proc IEEE Trans. on Control Syst. Techn.*, 26(3):1083–1090.

[23] Plumet, F., Saoud, H., and Hua, M.-D. (2013). Line following for an autonomous sailboat using potential fields method. In *Proc MTS/IEEE, OCEANS-Bergen*, pages 1–6.

[24] Sabău, S., Oară, C., Warnick, S., and Jadbabaie, A. (2017). Optimal distributed control for platooning via sparse coprime factorizations. *IEEE Transactions on Automatic Control*, 62(1):305–320.

[25] Saoud, H., Hua, M.-D., Plumet, F., and Amar, F. (2014). Modeling and control design of a robotic sailboat. In *Robotic Sailing 2013*, pages 95–110.

[26] Saoud, H., Hua, M.-D., Plumet, F., and Amar, F. B. (2015). Routing and course control of an autonomous sailboat. In *Proc IEEE ECMR*, pages 1–6.

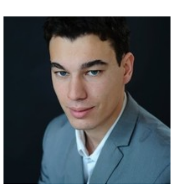
[27] Stelzer, R., Proll, T., and John, R. (2007). Fuzzy logic control system for autonomous sailboats. In *In IEEE Fuzzy Systems Conference*, pages 1–6.

[28] Utkin, V. (2013). *Sliding modes in control and optimization*. Springer Science & Business Media.

[29] Viel, C., Vautier, U., Wan, J., and Jaulin, L. (2018). Position keeping control of an autonomous sailboat. *IFAC CAMS*, 51(29):14–19.

[30] Xiao, L. and Jouffroy, J. (2014). Modeling and nonlinear heading control of sailing yachts. *IEEE Journal of Oceanic engineering*, 39(2):256–268.

[31] Zheng, Y., Li, S. E., Li, K., and Ren, W. (2018). Platooning of connected vehicles with undirected topologies: Robustness analysis and distributed h-infinity controller synthesis. *IEEE Transactions on Intelligent Transportation Systems*, 19(5):1353–1364.



Christophe Viel received master degree in ”Advanced System and Robotic” and an engineering degree at the Arts et Metier ParisTech in Paris (France) in 2014. He received his Ph.D at the ONERA (The French Aerospace Lab) on the topic ”Determination of command control and observator for multi-agent system with reduced information by event-triggered” in 2017. Currently he is in a post-doctorate at the University of Plymouth in UK. His research topic is controls and strategies for autonomous sailboat.

Ulysse Vautier is a PhD student in Autonomous Sailboats. He is specialized in robotics, with expertise in signal processing and autonomous navigation systems. He received his engineering degree in Polytech, Paris, France and a Research Master Degree in Intelligent System - Signal Processing at Pierre & Marie Curie University, Paris. He worked at Blue Frog Robotics, a startup making a commercial companion robot. He is now doing his PhD at University of Plymouth.

Jian Wan received the B.Eng. and M.Eng. degrees from Northwestern Polytechnical University, Xi’an, China, in 1997 and 2000, respectively, and the Ph.D. degree in control engineering from the University of Girona, Catalonia, Spain, in 2007. He was a Research Assistant with the University of Leeds, U.K., from 2007 to 2009, a Research Associate with the University of Manchester, U.K., from 2009 to 2011, a Research Fellow with Maynooth University, Ireland, from 2012 to 2014, and also a Research Associate with the University of Strathclyde, U.K., in 2014. He joined the University of Plymouth, U.K., as a Lecturer in control systems engineering, in 2015. His research interests include statistical processing monitoring and control, set-membership methods for control, constrained optimization and control with applications to manufacturing processes, renewable energy, robotics, and autonomous systems.

Luc Jaulin received the Ph.D. degree in automatic control from the University of Orsay, France in 1993. He is currently Professor of Robotics at the ENSTA Bretagne, Brest, France since 2004. He does his research on underwater robotics using interval methods and constraint propagation. He was awarded the Moore prize in 2012 for application to interval methods to robotics.